$8a+12b$, $288a+408b$, etc. By adding a to each term of each series we have two series of the value of x. These series hold good when either a or b is zero; but if both are zero, $x=0$.

It will be noticed that this solution applies the terms of the question to the expression $2x^2+2ax+b=\square$, the value of x in the latter being a less than in the former.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.**

$2x^2-2ax+b^2=0$. $\therefore x=\frac{1}{2}(a\pm\sqrt{a^2-2b^2})$.

Let $a=p^2+2q^2$, $b=2pq$. Then $x=p^2$ or $2q^2$.

$\therefore$ p	q	a	b	x,
2	1	6	4	4 or 2,
3	2	17	12	9 or 8,
4	3	34	24	16 or 18,
1	2	9	4	1 or 8,
etc.	etc.	etc.	etc.	etc.

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

54. **Proposed by HENRY HEATON, M. Sc., Atlantic, Iowa.**

A man is at the center of a circle whose diameter is equal to three of his steps. If each step is taken in a perfectly random direction, what is the probability, (1), that he will step outside the circle at the second step, and, (2), that he will step outside at the third step?

I. **Solution by the PROPOSER.**

Let O be the center of the circle, A, the end of the first step, and B, the end of the second, and C, the end of the third.

Let $\angle OAB=\theta$, $\angle OBC=\phi$, $OB=x$, and $OC=y$.

Then if the length of the step be taken as the unit of measure, $x=2\sin\frac{1}{2}\theta$, and $y=(x^2+1-2x\cos\phi)^{\frac{1}{2}}=(4\sin^2\frac{1}{2}\theta+1-4\sin\frac{1}{2}\theta\cos\phi)^{\frac{1}{2}}$.

If $x=\frac{3}{2}$, B falls upon the circumference of the circle, and $\theta=2\sin^{-1}\frac{3}{4}$. If θ be $>2\sin^{-1}\frac{3}{4}$, and $<\pi$, the second step falls outside the circle. The probability of this is $P_1=(\pi-2\sin^{-1}\frac{3}{4})/\pi$.

If θ be $<2\sin^{-1}\frac{3}{4}$, and $y=\frac{3}{2}$, C falls upon the circumference of the circle, and $4\sin^2\frac{1}{2}\theta+1-4\sin\frac{1}{2}\theta\cos\phi=\frac{9}{4}$ or $\phi_1=\cos^{-1}(\sin\frac{1}{2}\theta-5/16\sin\frac{1}{2}\theta)$. Hence if ϕ be $>\phi_1$ the third step falls outside the circle. The chance that ϕ will be $>\phi_1$ and $<\pi$ is $(\pi-\phi_1)\pi$. The chance that θ has any particular value is $d\theta/\pi$. Hence the probability that the third step falls outside the circle is

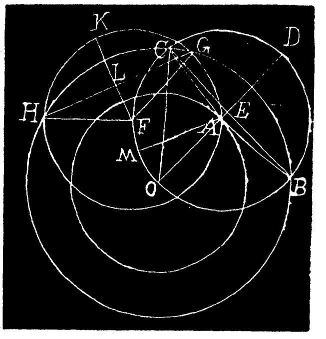
$$P_2=\frac{1}{\pi^2}\int_0^{2\sin^{-1}\frac{3}{4}}(\pi-\phi_1)d\theta.$$

This is not integrable in general terms but its value may be readily approximated by methods of mechanical quadrature.

II. Solution by G. B. M. ZERR. A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va., and J. A. MOORE. Ph. D.. Professor of Mathematics, Millsaps College. Jackson, Miss.

Let $AO=a$, then $CO=\frac{3}{2}a$.

(1). Let his first step place him at the point A, then in order that he may step outside on the second step he must step somewhere on the arc CDB.



Let $AE=t$, $EC=u$, $\angle DAC=\beta$, P=chance in (1), p=chance in (2).

Now $u^2=a^2-t^2=\frac{9}{4}a^2-(a+t)^2$, $\therefore$ $t=\frac{1}{8}a$.

$\therefore$ $\cos\beta=\frac{1}{8}$.

$\therefore$ $P=\beta/\pi=\cos^{-1}\frac{1}{8}/\pi=.460106$.

(2). Let chord $OM=\frac{1}{2}a$, then in order that he may step out the third step he must step somewhere on the arc CM or its equal on the opposite side

$$\angle CAM=\delta=\pi-(\beta+OAM)$$

$$=\cos^{-1}\left(\frac{3\sqrt{105}-7}{64}\right).$$

P_1=chance he steps on this arc$=\delta/\pi=.379034$.

If his second step places him on arc CM then his third step must place him on the arc GKH. The $\angle KFH$ may vary from 0 to $\cos^{-1}(-\frac{1}{3})$.

$\therefore$ p_1=chance that he steps on arc $GKH=\dfrac{\cos^{-1}(-\frac{1}{3})}{2\pi}$.

$\therefore$ $p_1=.304086$.

Now $p=P_1\times p_1=\delta/\pi\times\dfrac{\cos^{-1}(-\frac{1}{3})}{2\pi}=.115259$.

Solved with a different result by *CHAS. C. CROSS*.

55. Proposed by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

It has been clear for 15 consecutive days, what is the chance of the 16th day being cloudy ?

Solution by the PROPOSER.

Let p=chance, p_1=chance that 16th day is clear.

$$\therefore p_1=\frac{\int_0^1 x^{16}dx}{\int_0^1 x^{15}dx}=\tfrac{16}{17}.\quad \therefore p=1-p_1=\tfrac{1}{17}.$$